A Technique for Estimating Snow Transport Rate from the Mass Flux at a Given Height

Masaru MATSUZAWA and Satoshi OMIYA
Civil Engineering Research Institute for Cold Region,
Public Works Research Institute, Japan
Introduction

Snow transport rate ($Q$): the mass of blowing snow particles that pass through a unit width per unit time ($g \, m^{-1} \, sec^{-1}$)

When planning and designing blowing-snow control facilities, it is necessary to know the cumulative annual snow transport.

Tabler (1994)
Introduction

The snow transport rate \((Q)\) is generally estimated by using empirical equations.

- Budd et al. (1966) \( \log Q = 1.22 + 0.0859 V_{\downarrow 10} \)
- Tabler (2003) \( Q = 0.00428 V_{\downarrow 10}^{\uparrow 3.8} \)
- Takeuchi (1980) \( Q = 0.0029 V_{\downarrow 1}^{\uparrow 4.16} \)
- Kobayashi (1972) \( Q = 0.03 (V_{\downarrow 1} - 1.3)^3 \)
- Matsuzawa et al. (2010) \( Q = 0.05 V_{\downarrow 1.2}^4 \)

\( V_x \): wind speed at the height of \( x \) (m)

Accurate determination of actual snow transport rate by using wind speed is difficult.
Mass flux and the snow particle counter (SPC)

**Mass flux (q)**: mass of the snow particles that pass through a unit area per unit time (g m\(^{-2}\) sec\(^{-1}\))

An SPC can measure mass flux continuously.

The SPC optically measures the number and size of airborne snow particles that pass between transmitter and receiver.

To clarify the relationship between the mass flux at a given height and the snow transport rate of blowing snow.
Field observation overview

The mass flux of snow was measured at the heights of 0.02, 0.05, 0.07 and 0.1 m using a box-shaped net-type blowing snow trap, and at the heights of 0.1, 0.3, 0.5, 1.0 and 2.0 m using a cylindrical net-type blowing snow trap.
## Meteorological data for the days of measurement

<table>
<thead>
<tr>
<th>Date</th>
<th>Air temperature (°C)</th>
<th>Wind speed at the height of 1 m (m/s)</th>
<th>Precipitation (mm/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feb. 21, 2012</td>
<td>-4.9 ~ -6.5</td>
<td>9.2 ~ 12.9</td>
<td>0.0</td>
</tr>
<tr>
<td>Feb. 3, 2013</td>
<td>-6.3 ~ -7.8</td>
<td>9.7 ~ 10.5</td>
<td>0.0</td>
</tr>
<tr>
<td>Jan. 31, 2014</td>
<td>-4.9 ~ -5.9</td>
<td>6.1 ~ 12.5</td>
<td>0.0 ~ 1.2</td>
</tr>
<tr>
<td>Mar. 6, 2014</td>
<td>-4.8 ~ -5.9</td>
<td>5.6 ~ 7.7</td>
<td>0.0 ~ 0.6</td>
</tr>
</tbody>
</table>
Calculation method to determine snow transport rate from mass flux of snow
Results

The relational equation $Q = kq$ ($k$ is a proportional constant) was obtained for the height of 0.5 m and the height of 1.0 m.

\[ Q = 3.0 \, q_{0.5} \]
\[ q_{0.5} \text{: Mass flux at 0.5 m height} \]

\[ Q = 5.4 \, q_{1} \]
\[ q_{1} \text{: Mass flux at 1.0 m height} \]
Discussion

Estimation model of snow transport rate

Modes of transport

Wind

$Q_{total} \equiv Q_{sal} + Q_{sus}$

Saltation $Q_{sal}$

Creep $\sim 0.01m$

Suspension $Q_{sus}$

$0.1 \sim 100m.$

Takeuchi (1990)
**Estimation model of snow transport rate,**

1) **Q in the suspension layer:** $Q_{sus}$

\[
q(z) = N(z) \cdot V(z) \quad \cdots (1)
\]

\[
Q_{sus}(z) = \int q(z) dz = \int N(z) \cdot V(z) dz \quad \cdots (2)
\]

\[
N(z) = \frac{P}{w_f} + \left( N_t - \frac{P}{w_f} \right) \left( \frac{z}{Z_1} \right)^{-\frac{w_b}{ku_*}} \quad \cdots (3)
\]

\[
V(z) = \frac{u_*}{k} \ln \left( \frac{z}{Z_0} \right) \quad \cdots (4)
\]

where, \( N(z) \): blowing snow density, \( V(z) \): wind velocity,

\( P \): precipitation intensity, \( N_t \): blowing snow density at reference height \( z_1 \),

\( w_f \): falling speed of snowfall particles,

\( w_b \): falling speed of suspended particles

\( k \): Karman coefficient (0.4), \( u_* \): friction velocity, \( z_0 \): surface roughness

Discussion (cont’)

Matsuzawa and Takeuchi (2002)
1) \( Q \) in the suspension layer: \( Q_{\text{sus}} \)

\[
q(z) = \frac{P u_*}{k w} \left( \ln z - \ln z_0 \right) + \frac{a}{z_1^b} \cdot z^b \ln z - \left( \frac{a}{z_1^b} \ln z_0 \right) \cdot z^b
\]

where, \( a = \left( N_t - \frac{P}{w_f} \right) \frac{u_*}{k} \quad b = - \frac{w_{b}}{ku_*} \)

\[
Q_{\text{sus}} = \left[ \frac{P u_* z}{k w_f} \left( \ln \frac{z}{z_0} - 1 \right) + \frac{a}{b + 1} \frac{z_{b+1}}{z_1^b} \left( \ln \frac{z}{z_0} - \frac{1}{b + 1} \right) \right]^{z_2}_{z_1}
\]

2) \( Q \) in the saltation layer: \( Q_{\text{sal}} \)

\[
Q_{\text{sal}} = 0.03(V_1 - 1.3)^3
\]

\[
Q = Q_{\text{sal}} + Q_{\text{sus}}
\]

Discussion (cont’)

Kobayashi (1972)
**Calculation**

\[
Q_{\text{sus}} = \left[ \frac{Pu*z}{kw_f} \left( \ln \frac{z}{z_0} - 1 \right) + \frac{a}{b+1} \frac{z^{b+1}}{z_1^b} \left( \ln \frac{z}{z_0} - \frac{1}{b+1} \right) \right]^{z_2}_{z_1}
\]

Where,

\[
a = \left( N_t - \frac{P}{w_f} \right) u_* \\
b = -\frac{w_b}{ku_*}
\]

\[
Q_{\text{sal}} = 0.03(V_1 - 1.3)^3
\]  \( \cdots (6) \)

The following values are assigned to equation (6).

\[
z_0 = 1.5 \times 10^{-4} \text{ (m)} \, , \, w_f = 1.2 \text{ (m/s)} \, , \, w_b = 0.25 \text{ (m/s)} \\
z_1 = 0.15 \text{ (m)} \, , \, z_2 = 5.0 \text{ (m)} \\
u_* = 0.036 \, V_{10} \text{ (m/s)} \\
n_N = 0.021 \exp(0.401 \, V_{10}) \text{ (g/m}^3) \]

**Discussion**

Cases for calculation of \( Q \)

**Precipitation:** 0, 1, 2 (mm/h)
**Wind velocity (\( V_{10} \)):** 5.7 – 16.7 (m/s)
Mass flux of snow at the height of 0.5 m ($q_{0.5}$) vs. snow transport rate rate $Q$

From observed data:

$$Q = 3.0 \cdot q_{0.5}$$

The calculated values are roughly the same as the observed values.

$P$ is precipitation (mm/h).
Summary

• The following relational equations between the snow transport rate $Q$ and the mass flux $q$ at a given height were obtained from field observations.

\[ Q = 3.0 \, q_{0.5} \quad Q = 5.4 \, q_1 \]

Where,
\[ q_{0.5}: \text{mass flux of blowing snow at the height of 0.5m} \]
\[ q_1: \text{mass flux of blowing snow at the height of 1 m} \]

• Additionally, a model for estimating the snow transport rate from the mass flux was derived.

• The calculated values are found to be roughly the same as the observed values.

• It is considered that the empirical equations obtained from this study can be used to estimate the snow transport rate.
Thank you for your attention!