Forecasting of traffic jams on high-ways caused by adverse weather

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ABSTRACT

Adverse weather generally decreases visibility and increases slipperiness that can cause development of traffic jams on high-ways. The article presents a new model for prediction of the jam length based upon information about a proper speed limit at the critical road sector. For the prediction the model also applies a time series of traffic flow at this sector that is forecast by an intelligent module based upon records of traffic activity in the past. The forecast flow is converted to traffic jam characteristic by integrating the number of cars that are stopped due to decreased road capacity while more detailed description of the jam properties is given by the solutions of partial differential equations. Performance of the corresponding computer program is demonstrated by forecasting the evolution of the traffic jam during rush-hour at the point of maximal traffic activity of high-ways at Ljubljana in Slovenia where inconvenient influences of weather are often observed.

Keywords: forecasting, traffic jam, driving conditions

1   INTRODUCTION

At the previous Sirwec conference we have explained how parameters of driving conditions during severe weather can be predicted by a non-parametric statistical model.[1] Since driving conditions significantly influence traffic flow, our next aim is to show how the values of predicted parameters can be utilized at the forecasting of severe weather consequences. Here we pay attention to increased slipperiness and decreased visibility since they frequently disturb the traffic. Quite often the corresponding disturbances lead to traffic jams, therefore our goal is to formulate a mathematical model by which the jam properties can be forecast. For this purpose we apply three components by which the properties of population activity, traffic flow rate and driving conditions can be described. These components are joined in a more comprehensive model by which the evolution of traffic jam at a critical road sector can be forecast. By the first component the expected traffic flow rate at normal driving conditions is forecast for the considered road sector. The second component is the fundamental diagram that yields the road capacity. The most important parameter of the fundamental diagram is the speed limit. It is estimated from the friction coefficient or visibility length by a characteristic specified in the third component. To describe the evolution of a jam we assume, that the population activity determines the traffic flow to the critical region. This flow is there partially stopped due to decreased road capacity that is caused by inconvenient driving conditions. By integrating the stopped flow we then proceed to simple estimation of jam properties, while more detailed description is performed by partial differential equations of traffic flow. In the following section the properties of the model components are briefly explained, while in the successive ones their incorporation into the new model and its applications are described.

2   COMPONENTS OF THE MODEL

2.1   Forecasting of traffic flow

The first component is an intelligent module developed for prediction of the traffic flow field $Q(x,t)$ on the high-way network in Slovenia.[2-4] Its graphic user interface is shown in Fig. 1. At its application a user selects the date, hour and point of observation. The corresponding data are used as conditions in the incorporated computer program when estimating the traffic flow field during the selected day on the complete network of observation

points. For this purpose the non-parametric statistical forecasting is performed by the conditional average estimator. Estimation is carried out based upon a representative set of traffic field records measured in the past. The evolution of the forecast field is presented by dynamic simulation at the top window of the module of Fig. 1. The position of the selected point is indicated by a vertical and horizontal line. The radius of the circle at a point of the graph indicates the amount of the flow rate at the selected time of forecasting. In addition to this, the dependence of traffic flow rate on time during the selected day and at the selected point is shown by the bottom graph. By the corresponding time series we describe the expected number of vehicles that are approaching the selected critical region where they are contributing to the formation of traffic jam due to inconvenient driving conditions caused by severe weather. For our demonstration we consider the traffic flow at the high-way point near Ljubljana where maximal activity is usually observed. This point is selected since there the moisture from the surrounding swamp often influences driving conditions either by increasing the slipperiness or decreasing the visibility distance due to fog. Two expressive peaks in the record of flow rate denote increased population mobility during rush-hours. Due to this property the recorded function can be also analytically described by the superposition of two normal distributions and a constant.[2] The corresponding graph is shown in Fig. 2.
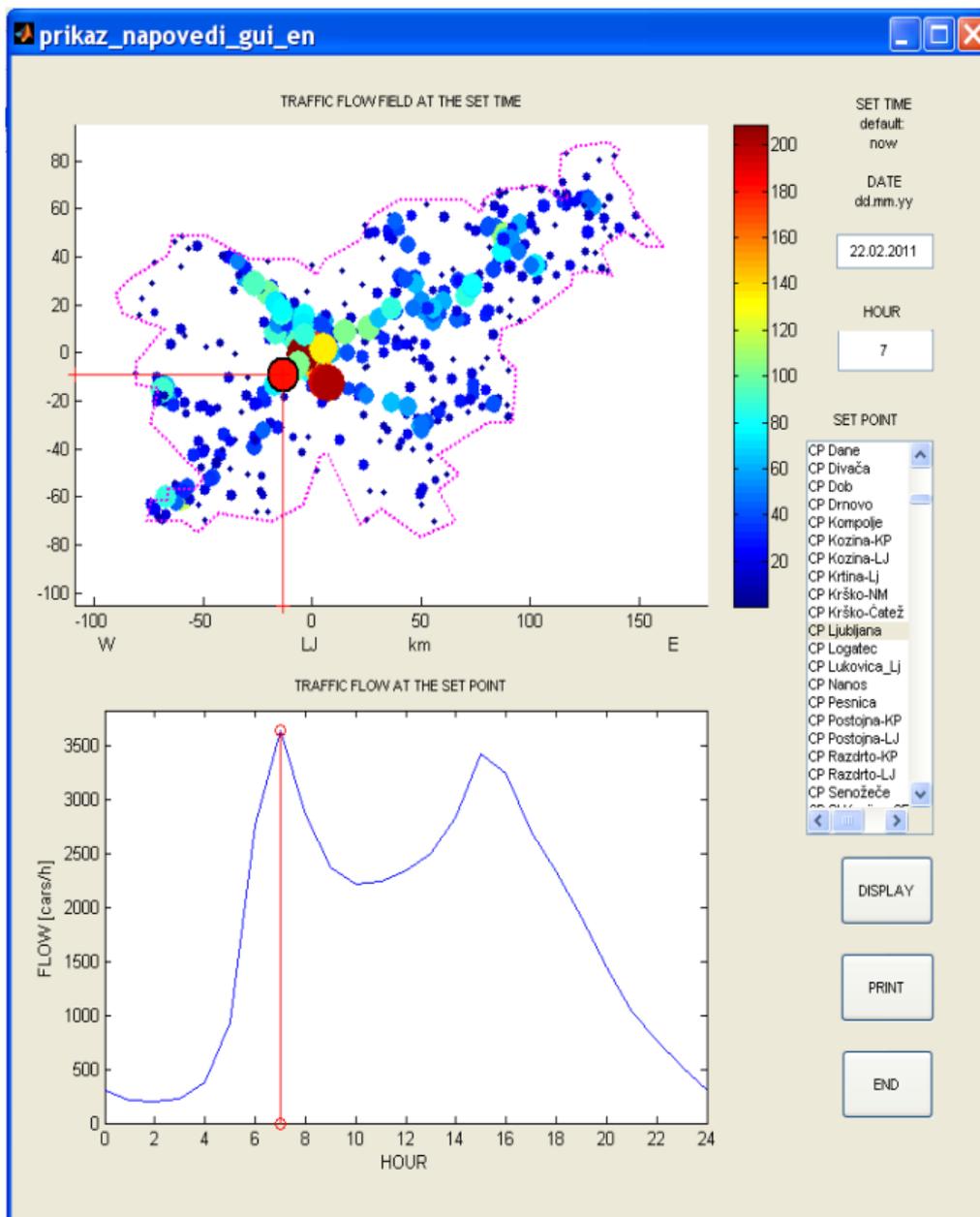

Figure 1 - The graphic user interface of the intelligent module for traffic flow forecasting. The top graph shows the distribution of the predicted traffic flow rate $Q(x)$ in Slovenia at set time, while the bottom one shows the dependence of $Q(t)$ at the selected position denoted by vertical and horizontal line in the top diagram. The radius of the circle at a point in this diagram indicates the value of flow rate at the time of forecasting.
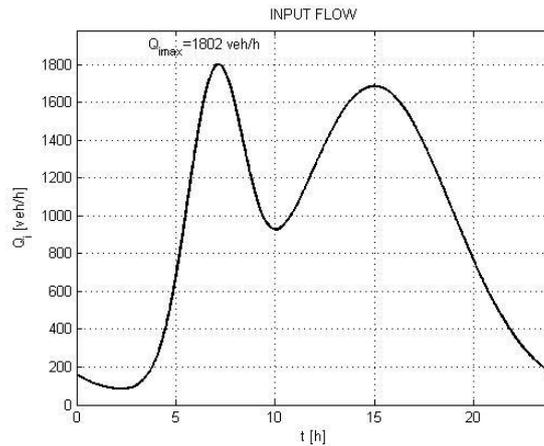
Figure 2. Analytically described dependence of the predicted traffic flow rate Q on time t. Data correspond to the flow on a single lane as forecast by the intelligent module and shown by the graphic user interface in Fig.1.

2.2 Fundamental diagram of traffic flow

Disturbances of traffic flow that are caused by increased slipperiness or decreased visibility can be most simply described by a drop of allowed speed limit on the road. Its influence on the traffic flow is described by the fundamental diagram of traffic flow that is briefly formulated in the following paragraphs. This diagram is then used in the description of the road capacity at the critical region that is the basic characteristic for estimation of the jam growth.

At the formulation of the fundamental diagram we use a macroscopic description based upon the mean density ρ and velocity v of cars.[5,6] The graph of the relation $v=v(\rho)$ represents the fundamental diagram of traffic.[6] The corresponding flow rate is expressed by $Q=\rho v$ and represents the basic variable for the description of the traffic state in the critical region. By using this expression we can transform the first fundamental diagram into the second one that shows the flow rate as a function of density: $Q(\rho)$. To formulate this function we consider quasi-static and homogeneous free flow of vehicles on a high-way with the speed limit v_o. The density ρ is determined by the distance between cars r as: $\rho = 1/r$. Drivers adjust distance between cars so that it grows with velocity: $r = \lambda + \tau w$. Here λ denotes the mean length of cars, τ the reaction time, and w a characteristic or *proper* velocity determined by the transition of the clear spacing between cars $r - \lambda$ in the reaction time τ. For our purpose we change above relation to: $w(\rho) = (r - \lambda)/\tau = (1/\rho - \lambda)/\tau$. On average a driver tries to keep the velocity of car v bellow the characteristic value w, and also bellow the limit v_o. By considering w and v_o as joint constraints we have found that empirical relation $v(\rho)$ can be surprisingly well described by the function:

$$v = v_o /(1+ u \, v_o /w^2)= v(\rho) \tag{1}$$

By expressing the parameter as $u = C\lambda/\tau$ and using values: $C = 3$, $\lambda = 4.4$m and $\tau = 1.3$s we adapted our law to experimental data given in ref. [5] Since the proper velocity w depends on density of cars ρ, Eq. 1 describes the fundamental traffic law $v = v(\rho)$ and its first fundamental diagram. On the left side of Fig. 3 the agreement between experimentally and theoretically determined first diagram is shown by the middle two curves (---, -o-) for the case with the limit value v_o=110km/h. Similarly, the corresponding second diagram for the flow rate $Q(\rho)$ is shown on the right side of Fig. 3. Experimental data are taken from the ref. [5] and correspond to a high-way with the speed limit 120km/h that is decreased to the value 110km/h due to the presence of trucks. The limit value v_o = 110km/h has been selected to render possible comparison with models of other authors.[5,6] In Slovenia the velocity limit on normal high-ways is v_{o1} = 130km/h while frequently observed limit on a disturbed road is v_{o2}=60 km/h. Diagrams corresponding to them are also presented in Fig. 3 by the top and bottom curves.

The maximal value of the flow rate determines the *road capacity*. Its dependence on the speed limit has been determined numerically from the second fundamental law and is shown in Fig. 4. The capacity is decreasing with the decreasing speed limit. For a single lane the capacities at the limit values v_{o1}=130km/h and v_{o2}=60km/h are $Q_{max1} \sim 2.2*10^3$veh/h and $Q_{max2} \sim 1.4*10^3$veh/h respectively. A jam appears when the flow to a disturbed road sector reaches its capacity.

2.3 Speed limit corresponding to driving conditions in severe weather

The influence of a severe weather on traffic flow can be most simply described by a proper speed limit v_o that is smaller than v_{o1}. To estimate it we consider the car *stopping distance* x_{st} at the speed limit that is comprised of the *reaction* distance x_r and the *braking* distance x_b.[5-8] The first one is determined by the velocity v and time τ
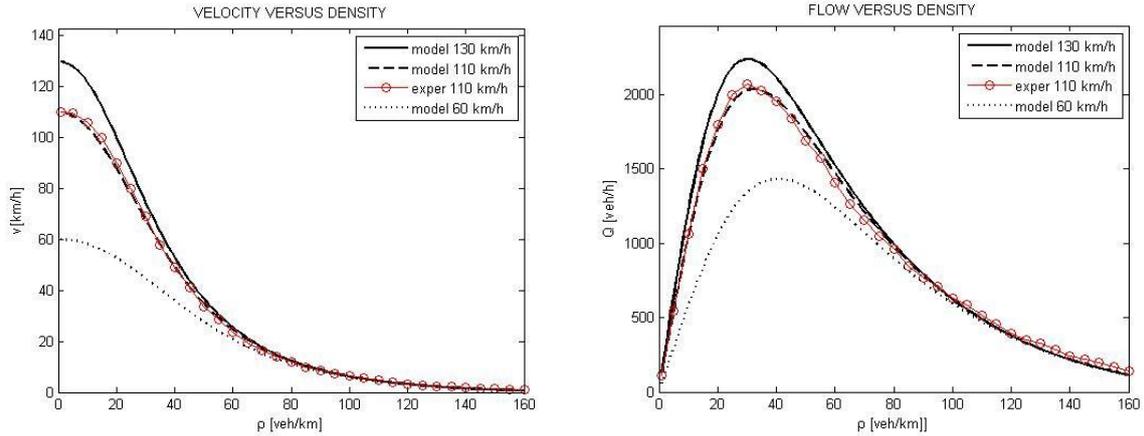
Figure 3. Dependence of velocity v and traffic flow rate Q on the density of cars ρ.
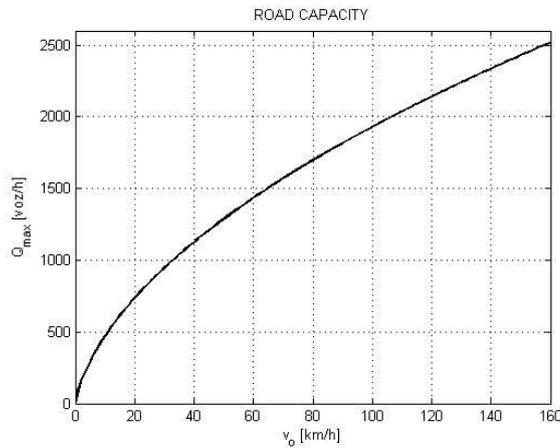Experimental data are taken from the ref. [5].

Figure 4. Dependence of the road capacity Q_m on the speed limit v_o.

of reaction: $x_r = v\tau$ and describes the distance that a car drives before it starts barking. The second is determined by the dynamics of the vehicle that is governed by the control of braking mechanism and the friction of wheels on the pavement. We consider just the friction and describe it by the friction coefficient μ that is generally dependent on the velocity: $\mu=\mu(v)$. In a simple description this dependence is avoided by assuming just a constant average value that leads to quadratic dependence of braking distance on the speed limit v_o:

$$x_{st} = \tau v + v^2 /2\mu g \tag{2}$$

Here g=9.81m/s^2 denotes the acceleration of gravity. However, a simplified treatment can yield just a very rough estimate of stopping distance since the friction coefficient at high velocities, that correspond to speed limits, is drastically reduced.[7] Friction is a stochastic phenomenon depending on pavement and tire characteristics. Consequently, it is hard to describe univocally the function $\mu(v)$ although experimental data indicate [7] that the following exponential function properly describes its main features:

$$\mu(v)= \mu_o \exp(-v/c) \tag{3}$$

Here μ_o denotes the static value of friction coefficient and c the characteristic decay parameter with a typical value: c=85km/h. By integrating corresponding dynamic equations for v and x up to time of stopping we obtain:

$$x_{st} = \tau v + c^2 [1 + (v/c -1) \exp(v/c)] /2 \mu_o g \tag{4}$$

This equation shows that the decay of friction coefficient at increased velocities significantly increases the stopping distance. Numerical analysis reveals that the braking distance in Eq. (4) can be approximately expressed by the corresponding value in Eq. (2) as:

$$x_{st} \sim \tau v + \exp(0.7 \ v/c) \ v^2/2 \ \mu_o g \tag{5}$$

At given velocity v the first term is mainly determined by the alertness of a driver while the second term is determined by the slipperiness of the road surface that increases with decreasing friction coefficient. If a severe weather decreases friction, then we can provide for equal braking distance as during normal conditions at the speed limit v_{o1} by reducing the limit value to v_{o2}. By using Eq. (2) the relation $v_{o1}^2/2\mu_{o1}g = v_{o2}^2/2\mu_{o2}g$ is obtained which yields a simple expression for a reduced proper speed limit in dependence of the reduced constant μ_{o2}:

$$v_{o2} = v_{o1}\,(\mu_{o2}/\mu_{o1})^{1/2} \tag{6}$$

Similarly the second term in Eq. (5) yields:

$$\mu_{o2}/\mu_{o1} = (v_{o2}/v_{o1})^2 \exp[(v_{o2}/v_{o1}-1)\ 0.7\ v_{o1}/c] \tag{7}$$

Graphs of functions in Eqs. (6) and (7) are shown on the left side of Fig. 5. For dry high-quality pavement the value of friction coefficient μ_o is close to 0.9. This value is reduced to around 0.5 and below 0.2 on wet and icy pavement, respectively. Corresponding proper speed limits are reduced to ~90km/h and ~40km/h, respectively. By accounting the dependence of friction coefficient on velocity we obtain smaller values of proper speed limits due to increased braking distance. However, estimation of a proper speed limit from the braking distance is a bit questionable since it is based upon the complete velocity interval from the speed limit to zero. If we want to provide for similar driving conditions at the speed limit v_{o2} as in the case with v_{o1}, then we have to equalize just the corresponding friction coefficients. This leads to the equation: $\mu_{o1}\exp(-v_{o1}/c) = \mu_{o2}\exp(-v_{o2}/c)$ that yields:

$$v_{o2} = v_{o1} - c\,\ln(\mu_{o1}/\mu_{o2}) \tag{8}$$

The graph of this characteristic is also shown in Fig. 5 and shows much larger drop of proper speed limit with the increasing slipperiness as in the previous two cases. Not surprisingly, it indicates that the road should be in fact closed when icy conditions on the pavement are approached due to influences of a severe weather.

The similar approach can be used to treat a case with decreased visibility. The equation for the proper speed limit is obtained by setting the complete stopping distance in Eqs. (2) or (5) equal to the characteristic visibility distance d_v. The solution of the corresponding equations then yields the plots shown on the right side of Fig. 5. Accounting of friction coefficient dependence on the velocity yields also in this case much smaller speed limit value as in the case with constant μ_o.

In relation to application of characteristics shown in Fig. 5 there appears the question as how to estimate the friction coefficient and visibility distance from the forecast weather data. For this purpose one can follow the method described in ref. [1] or utilize some existing empirical relations.
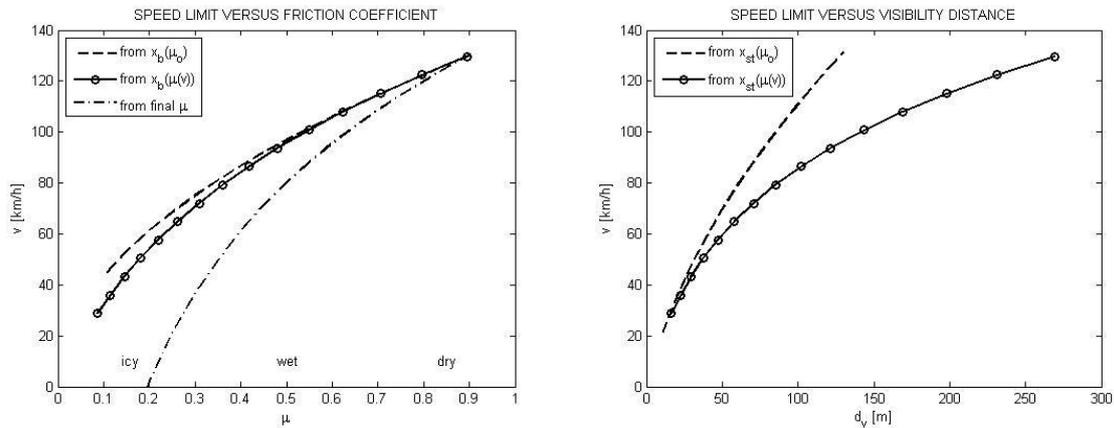

Figure 5. Dependence of the proper speed limit on friction coefficient (left) and visibility distance (right).

3   ESTIMATION OF JAM PROPERTIES

Our next goal is to demonstrate what would happen during the selected day with the traffic flow on the disturbed road at the previously mentioned point on the high-way near Ljubljana if the speed limit is decreased due to adverse weather conditions, e.g. from 130km/h to 60km/h. For this purpose we first try to estimate just the number of cars in jam ΔN by an approximate treatment and proceed with more strict description of the complete traffic field based upon partial differential equations in the next section.[9] With this aim we consider the case when the input flow Q_i is increasing with time. As long as the input flow Q_i is below the capacity of the disturbed road Q_{pmax} all cars pass fluently. But, when the input flow surpasses the capacity of the road, the difference $\Delta Q = Q_i - Q_{pmax}$ is stopped and causes evolution of jam in front of the disturbed road sector. From the dependence of input flow on time, we estimate the number of stopped cars ΔN by integrating $\Delta Q(t)$ with respect to time. The time that a car coming to the jam spends to pass it is then estimated by: $T_j = \Delta N/Q_{pmax}$. The jam

length Z can be estimated by the product of this time and the velocity of cars v_c in the critical region. At the maximal capacity it equals approximately one half of the speed limit $v_c \sim 0.5 v_{o2}$. The jam length can be thus estimated by the expression: $Z = 0.5 v_{o2} \Delta N / Q_{pmax}$.

To demonstrate applicability of above explained treatment we first determine the road capacity at the given value of the decreased speed limit from Fig. 4 and apply the forecast dependence of input flow shown in Fig. 2. By using these data we then obtain the graph on the left side of Fig. 6 that shows the input and the passed flow by dotted and solid lines, respectively. The latter is determined by the reduced capacity that determines the level of the horizontal section in the graph. The corresponding number ΔN of cars stopped in jam is shown in the right graph of Fig. 6. As said previously, the reduced speed limit does not permit all incoming cars during the rush-hour to pass the disturbed region when the input flow surpasses the reduced capacity of 1422veh/h. In the first peak the maximal value of stopped car number is $\Delta N_{max} \sim 550$. The corresponding waiting time and jam length are approximately: $T_j \sim 23$min, $Z \sim 13$km. The last value roughly corresponds to the value obtained from the traffic flow distribution determined by solving partial differential equations in the next section.[9]
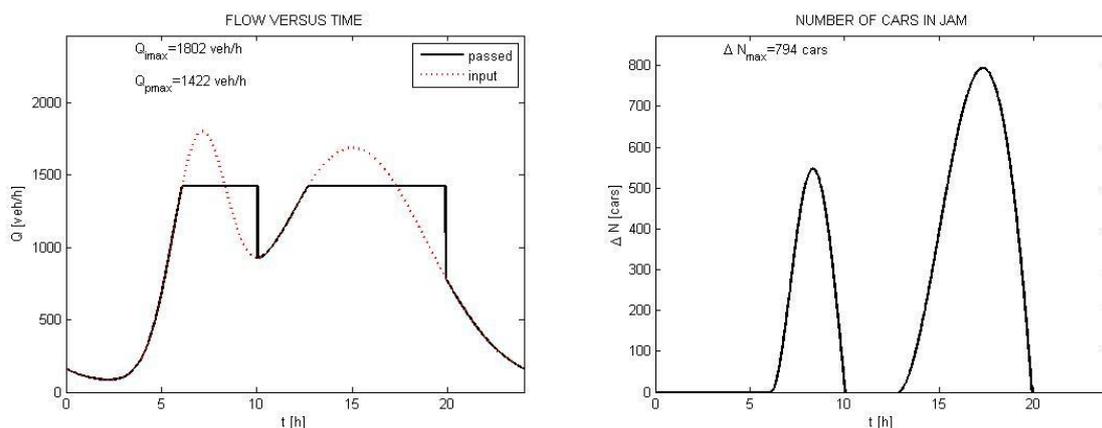

Figure 6. Dependence of the passed flow Q and the estimated number of cars in jam ΔN on time t.

Estimation of the jam length is less reliable than the corresponding number of cars, since the distance between cars is changing with their velocity that depends also on properties of the jam. However, the determination of the number of stopped cars is also only approximate since the jam can also influence the dynamics of the flow in the jam itself. More accurate determination of the jam length can thus be obtained just by a strict accounting of the flow dynamics which we briefly describe in the next section.[9,10] Irrespectively of this weakness, we can estimate the basic characteristics of jam by the integral of stopped flow that is the basic variable applicable for simple prediction of the jam evolution. For this purpose we just need to predict the input flow in dependence on time and to estimate the reduced capacity from weather conditions.[1,9]

5. DESCRIPTION OF THE JAM FIELD

In our derivation of the fundamental law Eq. (1) we considered quasi steady and homogeneous traffic state. However, this is not the case when treating evolution of traffic jams. Since the derived law Eq. (1) describes well the mean traffic properties in equilibrium, we further assume that the velocity at a certain position x and time t is adapted to the equilibrium value $v_e(\rho)$ determined by Eq. (1) during some characteristic adaptation time T.[6,10] We describe this adaptation by the most simple differential equation:

$$dv/dt = (v_e(\rho) - v)/T \tag{9}$$

and further consider the velocity and density as mutually dependent field variables $v = v(x,t)$ and $\rho = \rho(x,t)$. In accordance with this dv/dt in the differential Eq. (9) denotes the convective derivative: $dv/dt = \partial v/\partial t + v \partial v/\partial x$. The fundamental dynamic law of the field is then given by the continuity equation:[10] :

$$\partial \rho/\partial t + \partial(v\rho)/\partial x = I(x,t) \tag{10}$$

in which $I(x,t)$ denotes the traffic source term. If we start analysis at a certain point x_o where the traffic flow rate $Q(t)$ is forecast, then the source term can be described by the Dirac's delta function as: $I(x,t) = Q(t) \delta(x - x_o)$. Drivers try to adapt their velocity to the leading car with a delay specified by the reaction time τ. Consequently, when describing the adaptation of velocity v at the position x and time t, the density in Eq. (10) has to be taken at some position h ahead of x and at delayed time $t - \tau$. A typical value of h is several lengths of the car: $h \sim 3\lambda$. Similarly the relaxation time is several reaction times: $T \sim 3\tau$.

A solution of the system (1,9,10) can be found by standard numerical methods for treatment of partial differential equations. To demonstrate an example we assume that the critical sector takes place at $x \geq 22$km. The input flow is given by the function in Fig. 2 as in the simplified estimation of the jam length. Fig. 7 shows distributions of the traffic flow field calculated by using cell sizes of $\Delta x = 200$m and $\Delta t = 1$s. The flow enters the road section at $x=0$ and moves in the x direction as shown in Fig. 7a. At the rush-hour its amplitude first rises to the maximum and then falls again. When passing through the critical sector its maximal value is decreased and the peak is flattened. The reduction of the velocity in the sector is shown in Fig. 7b-right as a downward step. At low t the velocity is high at low x, but when cars pass the critical sector, their velocity is decreased due to the decreased speed limit. Simultaneously with decreasing velocity the density is increased as shown in Fig. 7c. With increasing time and flow at rush-hour the reduction of velocity in the sector leads to evolution of jam with an expressive peak. At the peak the jam exhibits wave-like structure that corresponds to stepwise movement of cars. When the rush-hour maximum is passed, the input flow again starts decreasing, which further leads to a decreased density, increased velocity and decay of the jam by the flow through the critical sector. A similar evolution of jam as in the morning is observed also in the afternoon rush-hour time. From the graphs of field variables we can directly read the forecast length of jam and its velocity of spreading.
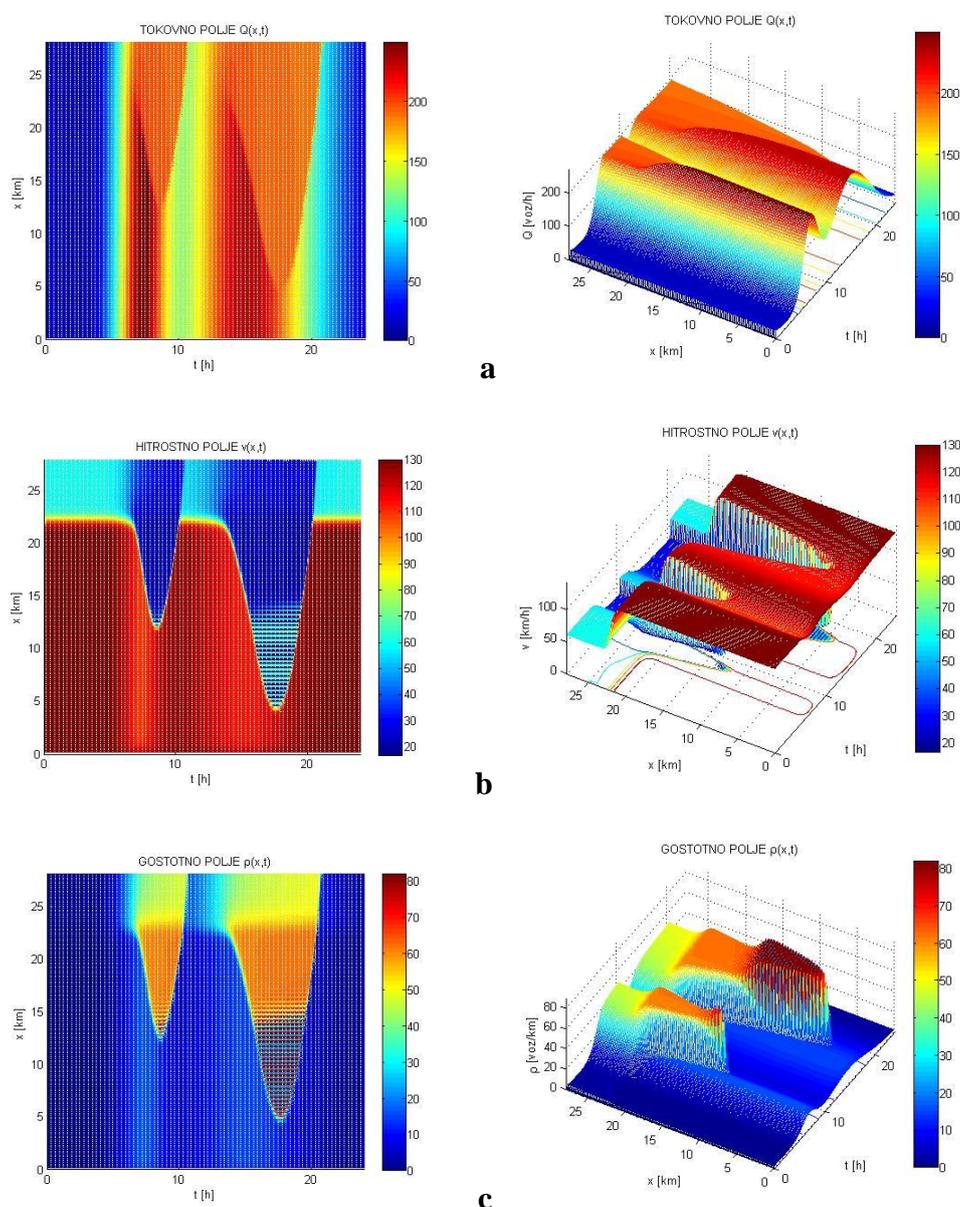
a

b

c

Figure 7. Distributions of traffic field variables: **(a)** flow rate, **(b)** velocity, **(c)** density.
Left – ground plan, right – side view.

4   CONCLUSIONS

We have shown that in spite of rather complex, non-linear, and stochastic character of traffic flow, it is possible to model the evolution of traffic jam at a disturbed region based upon the forecast input flow and the proper speed limit that corresponds to driving conditions in severe weather. For this purpose the flow rate predicted by the previously developed intelligent unit is readily applicable.[2-4] In addition, a proper speed limit can be estimated from the characteristics that represent the influence of friction coefficient and visibility distance on its value. However, these parameters have to be determined from forecast weather data by an appropriate method as is for example described in ref.[1] The second fundamental diagram provides for definition of road capacity and estimation of its value from the given speed limit. This further renders rather simple description of the traffic jam evolution and estimation of its parameters: ΔN, T_j, Z. An advantage is that for this purpose we need just the predicted input flow in dependence on time and the reduced capacity estimated from weather conditions.[1,9] However, more comprehensive is the description of jam evolution in terms of traffic field distributions based upon partial differential equations that yields more detailed insight into the properties of the complete phenomenon. The method presented here has been developed recently, and consequently, before practical applications it still needs a thorough experimental verification of performance in real situations.

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ACKNOWLEGEMENTS