# A Technique for Estimating Snow Transport Rate from the Mass Flux at a Given Height 

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1. Introduction

Snow transport rate is an index of blowing snow intensity. The snow transport rate is the mass of blowing snow particles that pass through a unit width per unit time. When planning and designing blowing-snow control facilities based on the assessment of the risks of blowing snow, it is important to know the cumulative annual snow transport rate (the total snow transport rate in one winter) ${ }^{1 \text { 1 }}$. The snow transport rate is generally estimated by using empirical equations whose variable is the wind velocity, because it is difficult to measure the snow transport rate during blowing snow events continuously. However, there are several different empirical equations developed by different researchers ${ }^{122) 3}$ ). The determined snow transport rate greatly depends on the choice of equation. Factors other than wind speed influence the snow transport rate (snowfall, for example). Even so, those factors have not been considered in the empirical equations. Furthermore, it is necessary to know the conditions under which blowing snow occurs, because there are cases in which the air temperature is high and blowing snow does not occur even though the wind speed would otherwise cause snow to blow. Accurate determination of actual snow transport rate by using wind speed is difficult.

The mass flux of blowing snow is the mass of snow particles passing through a unit cross-section per unit time. The value obtained by integrating the mass flux of blowing snow in the height direction equals the snow transport rate. A snow particle counter (SPC) is a device that can measure mass flux continuously. Use of SPCs has been increasing in blowing snow research. It would be possible to determine the cumulative annual snow transport rate based on continuous measurements, if the mass flux of blowing snow could be measured by installing a number of SPCs in a vertical array. However, such a technique is impractical, because SPCs are expensive. Therefore, the authors investigated the relationship between the mass flux at a given height and the snow transport rate to clarify the feasibility of measuring the cumulative snow transport with a single SPC.
2. The relationship between the mass flux of blowing snow and the snow transport rate as revealed by onsite observation

### 2.1 Measurement method

The authors measured the mass flux of blowing snow $q$ at several points of different heights at the Ishikari Blowing Snow Test Field, which is about 20 km north of Sapporo. At the site, cylindrical net-type blowing-snow trap meters and a box-shaped net-type blowing-snow trap meters were installed. The installation heights of the former were 0.1 $\mathrm{m}, 0.3 \mathrm{~m}, 0.5 \mathrm{~m}, 1.0 \mathrm{~m}$, and 2.0 m above the ground, and those of the latter were $0.02 \mathrm{~m}, 0.05 \mathrm{~m}, 0.07 \mathrm{~m}$ and 0.1 m above the ground. Each installation height was based on an opening in each meter (Figure 1). 10-min average wind speed at 1 m above the ground was measured. Precipitation was measured by using a rain gauge (resolution: 0.1 mm , 10-min measurement) installed on the ground and enclosed with a net that extended to a height of about 4 m . The
mass flux of blowing snow was measured 29 times in total on four days: February 21, 2012, February 3, 2013, January 31, 2014, and March 6, 2014. Among these measurements, precipitation was measured ten times. Table 1 lists the meteorological data for the days. The mass flux of blowing snow at the height of 5 m above the ground was determined from precipitation, unlike for the measurement of the mass flux of blowing snow at other heights. The snow transport rate $Q$ was calculated by adding up the mass fluxes of blowing snow measured at each height from 0.02 m to 5 m (Figure 2). The shaded area in Figure 2 corresponds to the snow transport rate.


Figure 1: Cylindrical net-type blowing-snow trap meters (left) and a box-shaped net-type blowing-snow trap meter (right)


Figure 2: Schematic showing the calculation method to determine snowdrift transport rate from mass flux of drifting snow

Table 1: Meteorological data for the days of measurement

| Date | Air temperature <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Wind speed at the <br> height of $1 \mathrm{~m}(\mathrm{~m} / \mathrm{s})$ | Precipitation (mm/h) |
| :---: | :---: | :---: | :---: |
| Feb. 21, 2012 | $-4.9 \sim-6.5$ | $9.2 \sim 12.9$ | 0.0 |
| Feb. 3, 2013 | $-6.3 \sim-7.8$ | $9.7 \sim 10.5$ | 0.0 |
| Jan. 31, 2014 | $-4.9 \sim-5.9$ | $6.1 \sim 12.5$ | $0.0 \sim 1.2$ |
| Mar. 6, 2014 | $-4.8 \sim-5.9$ | $5.6 \sim 7.7$ | $0.0 \sim 0.6$ |

### 2.2 Result

The relational equation $Q=\mathrm{kq}$ ( k is a proportional constant), which expresses the relationship between the snow transport rate $Q$ and the mass flux $q$, was obtained for the height of 0.5 m and the height of 1.0 m . The resulting relational equations were as follows: $Q=3.0 q$ for the mass flux at the height of 0.5 m , and $Q=5.4 q$ for the mass flux at the height of 1.0 m . The coefficients of determination for the equations are 0.86 and 0.71 , respectively (Figure 3 and 4).


Figure 3: Relationship between the mass flux of blowing snow at 0.5 m above the ground $q_{0.5}$ and snow transport rate $Q$.


Figure 4: Relationship between the mass flux of blowing snow at 1 m above the ground $q_{1}$ and the snow transport rate $Q$

## 3 Discussion

### 3.1 Estimation model of snow transport rate

Blowing snow particles are transported from one location to other locations by suspension, saltation and surface creep. Transportation by surface creep can be ignored for blowing snow, because it accounts for a negligible share of the total. Accordingly, the snow transport rate $Q$ can be considered as the sum of snow transport rate in the suspension layer $Q_{\text {sus }}$ and snow transport rate in the saltation layer $Q_{\text {sal. }}$. The following section describes a model to estimate the snow transport rate $Q$, and snow transport rate is calculated using the model.

The mass flux of blowing snow in the suspension layer $q$ is the product of blowing snow density $N(\mathrm{z})$, which is the blowing snow particle mass per unit volume, and the wind speed $V(z)$ (Equation (1)). The snow transport rate in the suspension layer $Q_{\text {sus }}$ is equal to the integral of the $q$ along the vertical direction (Equation (2)).

$$
\begin{align*}
& q(z)=N(z) \cdot V(z)  \tag{1}\\
& Q_{s u s}=\int q(z) d z=\int N(z) \cdot V(z) d z \tag{2}
\end{align*}
$$

The vertical profile of the blowing snow density in the suspension layer $N(\mathrm{z})$ is expressed as equation (3) ${ }^{4}$.

$$
\begin{equation*}
N(z)=\frac{P}{w_{f}}+\left(N_{t}-\frac{P}{w_{f}}\right)\left(\frac{z}{Z_{1}}\right)^{-\frac{w_{b}}{k u_{*}}} \tag{3}
\end{equation*}
$$

Where,
$N(\mathrm{z})$ is the blowing snow density at the given height $\mathrm{z}\left(\mathrm{g} / \mathrm{m}^{3}\right) . P$ is the snowfall intensity $\left(\mathrm{g} / \mathrm{m}^{2} / \mathrm{s}\right) . w_{\mathrm{f}}$ is the falling speed of snowfall particles ( $\mathrm{m} / \mathrm{s}$ ). $w_{\mathrm{b}}$ is the falling speed of blowing snow particles swirled from the snow surface (hereinafter: suspended particles) ( $\mathrm{m} / \mathrm{s}$ ). $z_{1}$ is the reference height (m) (the lower end of the suspension layer). $N_{\mathrm{t}}$ is the blowing snow density $\left(\mathrm{g} / \mathrm{m}^{3}\right)$ at the reference height $\mathrm{z}_{1} . k$ is Karman's constant $(=0.4)$. $u *$ is friction velocity ( $\mathrm{m} / \mathrm{s}$ ).

The wind speed profile is expressed as equation (4). Where, $\mathrm{z}_{0}$ is the surface roughness.

$$
\begin{equation*}
V(z)=\frac{u_{*}}{k} \ln \left(\frac{z}{z_{0}}\right) \tag{4}
\end{equation*}
$$

We substitute equations (3) and (4) into equation (1) to obtain equation (5).

$$
\begin{equation*}
q(z)=\frac{P u_{*}}{k w_{f}} \ln \left(\frac{z}{z_{0}}\right)+\frac{u_{*}}{k} \ln \left(\frac{z}{z_{0}}\right) \cdot\left(N_{t}-\frac{P}{w_{f}}\right)\left(\frac{z}{z_{1}}\right)^{-\frac{w_{b}}{k u_{*}}} \tag{5}
\end{equation*}
$$

Where,

$$
\begin{equation*}
a=\frac{u_{*}}{k}\left(N_{t}-\frac{P}{w_{f}}\right) \quad \cdots(6 \mathrm{a}), \quad b=-\frac{w_{b}}{k u_{*}} \tag{6b}
\end{equation*}
$$

When these two equations are substituted into equation (2), we obtain equation (7).

$$
\begin{equation*}
Q_{\text {sus }}=\frac{P u_{*} Z}{k w_{f}}\left(\ln \frac{z}{z_{0}}-1\right)+\frac{a}{b+1} \frac{z^{b+1}}{z_{1}^{b}}\left(\ln \frac{z}{z_{0}}-\frac{1}{b+1}\right)+c \tag{7}
\end{equation*}
$$

Where, $c$ is a constant of integration.
When equation (7) is integrated from $z_{1}$ to the upper end of the suspension layer, then the snow transport rate of the suspension layer $Q_{\text {sus }}$ is obtained.
In contrast, the snow transport rate of the saltation layer $Q_{\text {sal }}$ is determined by the relational equation below, obtained by Kobayashi ${ }^{5}$.

$$
\begin{equation*}
Q_{\text {sal }}=0.03\left(V_{1}-1.3\right)^{3} \tag{8}
\end{equation*}
$$

Where, $V_{1}$ is wind speed at the height of $1 \mathrm{~m}(\mathrm{~m} / \mathrm{s})$.
Based on the previous assumption, the snow transport rate $Q$ can be determined from the equation below.

$$
\begin{equation*}
Q=Q_{\text {sal }}+Q_{\text {sus }} \tag{9}
\end{equation*}
$$

3.2 Calculation results for the relationship between the snow transport rate and the mass flux of blowing snow at 0.5 $m$ above the ground

Next, the relationship between the mass flux of blowing snow at 0.5 m above the ground and the snow transport rate is determined. The values below are assigned to equations (6) and (7).

$$
z_{0}=1.5 \times 10^{-4}(\mathrm{~m}), w_{f}=1.2(\mathrm{~m} / \mathrm{s}), w_{b}=0.25(\mathrm{~m} / \mathrm{s}), z_{1}=0.15(\mathrm{~m}), z_{2}=5.0(\mathrm{~m})
$$

$u *$ is determined from wind speed by using equation (4), and $N_{t}$ is determined from the equation below ${ }^{6}$. Where, $V_{10}$ is wind speed at 10 m above the ground.

$$
\begin{equation*}
N_{t}=0.021 \cdot e^{0.401 V_{10}} \tag{10}
\end{equation*}
$$

From the above conditions, the snow transport rate and the mass flux of blowing snow were calculated for three cases of precipitation ( $0,1,2 \mathrm{~mm} / \mathrm{h}$ ), and the friction velocity ( $\mathrm{m} / \mathrm{s}$ ) was between $0.2-0.8 \mathrm{~m} / \mathrm{s}$ (equivalent to $4-18$ $\mathrm{m} / \mathrm{s}$ wind speed at the height of 1 m ).

Figure 5 shows the relationship between the mass flux of blowing snow at 0.5 m above the ground $q_{0.5}$ and the snow transport rate $Q$. $Q$ was found to increase with increase in $q_{0.5}$. The effect of the differences in precipitation on the calculation results was small. Figure 6 shows the calculation results that show the relationship between the friction velocity and the ratio of snow transport rate to the mass flux of blowing snow at 0.5 m above the ground $Q / q_{0.5}$. When
the friction velocity is $0.5 \mathrm{~m} / \mathrm{s}$ or greater, $Q / q_{0.5}$ correspond largely with the coefficient (3.0) of the relational equation obtained in Chapter 2. The snow transport rate is strongly dependent on wind velocity, and it is considered that the snow transport rate during weak winds (i.e., when the friction velocity is low) accounts for only a small portion of the cumulative snow transport rate. Therefore, it is considered that the relational equation obtained in Chapter 2 can be used to estimate the cumulative snow transport rate.


Figure 5: Relationship between the mass flux at the height of $0.5 \mathrm{~m} q_{0.5}$ and the snowdrift transport rate $Q$ ( P is precipitation ( $\mathrm{mm} / \mathrm{h}$ ).)


Figure 6: Relationship between the friction velocity u* and the ratio of snowdrift transport rate to the mass flux of drifting snow at the height of $0.5 \mathrm{~m} Q / q_{0.5}$ (P is precipitation (mm/h).)

## 4. Summary

A relational equation $Q=\mathrm{kq}$ ( k is a proportional constant), which expresses the relationship between the snow transport rate $Q$ and the mass flux $q$, was obtained for the height of 0.5 m and for the height of 1.0 m . Additionally, a model for estimating the snow transport rate was derived, and the calculated snow transport rate and the observed snow transport rate were compared. This comparison found that the calculated values were roughly the same as the observed values at friction velocities of $0.5 \mathrm{~m} / \mathrm{s}$ or greater. As a result, it is considered that the relational equation obtained from observation can be used to estimate the snow transport rate.

## References

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